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**NOMOGRAPHS FOR PARAMETRIC TRANSMITTING  
ARRAY CALCULATIONS**

**James C. Lockwood**

**Texas University**

**Prepared for:**

**Naval Ship Systems Command**

**6 February 1973**

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James C. Lockwood

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NAVAL SHIP SYSTEMS COMMAND  
Contract N00024-72-C-1380,  
Proj. Ser. No. SF 11111500, Task 16200



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Source level						
Beamwidth						
Graphical solution						
Shock threshold						

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## Introduction

A set of nomographs that facilitate calculations of parametric array farfield source level and beamwidth has been developed. These nomographs are an aid for applying the farfield parametric array theory as formulated by Berkay and Leahy.<sup>1</sup> The results obtained by use of this theory are strictly valid only in cases when finite amplitude attenuation of the primary wave is not significant. The nomographs may be used with considerable confidence for cases in which the primary source level is such that shocks will never form. A graph of the levels below which shocks will not form is provided.

A graphical aid for parametric array design has been given previously by Mellen and Moffett.<sup>2</sup> The Mellen-Moffett curves are more general than the present nomographs in that they include cases in which finite amplitude attenuation is important. However, for cases in which the present nomographs are applicable, their use is also much more direct. The numerical evaluation of parameters is avoided. Furthermore, for a given salinity, temperature, and pressure, a single set of nomographs may be used for a continuous range of primary and difference frequencies. The fact that saturation effects have not yet been taken into account in these nomographs is an obvious disadvantage. However, the Berkay and Leahy theory is applicable in many cases of interest, and these nomographs should prove useful for obtaining quantitative results quickly in such cases.

In the following pages a set of nomographs for sea water at 20°C is presented along with explanations of the equations they are designed to solve and detailed instructions for their use.

### The Nomographs and Instructions for Their Use

The solution given by Berkley and Leahy for the farfield source level and beamwidth of a parametric array formed by the two frequency radiation from a planar piston is formulated in terms of the Westervelt<sup>3</sup> solution for perfectly collimated primary radiation. The solution for an array formed by piston beams involves modifications of the Westervelt results that depend on the ratio of the primary beamwidths to the difference frequency beamwidth predicted on the basis of the Westervelt model.

Figure 1 is a nomograph from which the Westervelt source level and beamwidth may be obtained. The Westervelt half beamwidth is obtained by lining up the primary center frequency and the difference frequency and reading  $\theta_d$  in degrees. The nomograph is thus used to solve the equation

$$\theta_d = \sqrt{\left(2\alpha_T/k_d\right)} , \quad (1)$$

where

$\alpha_T$  is the sum of the primary absorption factors, and  
 $k_d$  is the difference frequency wave number.

The source level of a perfectly collimated array is obtained from fig. 1 by lining up the primary and difference frequencies and noting the intersection on axis X and then lining up the point on X so obtained with the value of  $L_{sd}$ -DI and reading the difference frequency source level  $L_{sd}$  in dB re 1  $\mu$ bar at 1 yd.  $L_s$  is the mean primary source level and DI is the directivity index of the source transducer

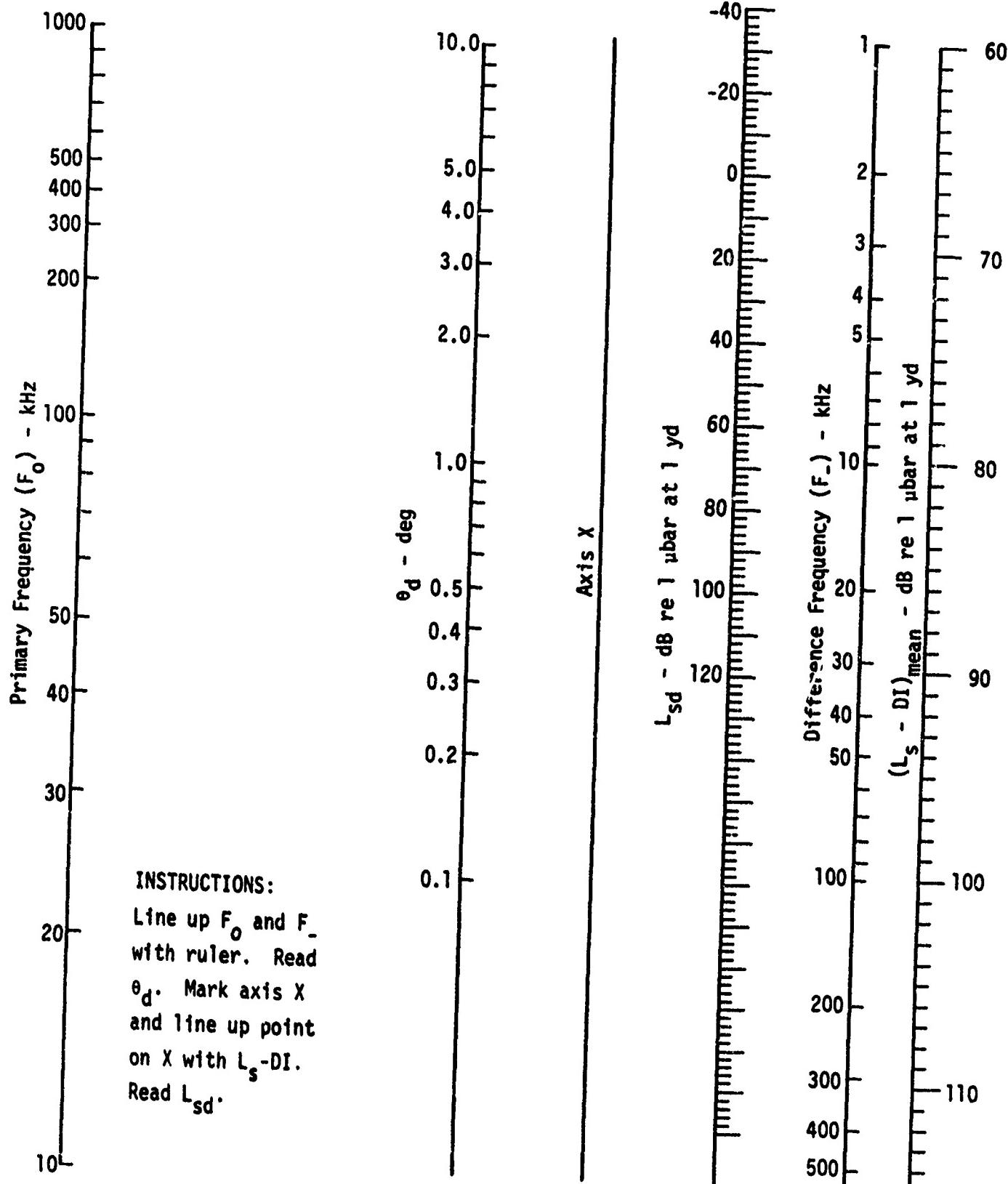


FIGURE 1  
NOMOGRAPH FOR DETERMINING  
DIFFERENCE FREQUENCY SOURCE LEVEL ( $L_{sd}$ ) AND  
HALF BEAMWIDTH OF A PARAMETRIC ENDFIRE ARRAY  
IN SEA WATER (WESTERVELT)

at the primary center frequency. The expression for difference frequency pressure upon which the solution is based is

$$p_- = \frac{\omega_-^2 \sqrt{W_1 W_2} \beta}{2\pi c_o^3 R \alpha_T} , \quad (2)$$

where

$\omega_-$  is the angular difference frequency,

$W_1$  and  $W_2$  are the total radiated powers at the two primary frequencies,

$\beta$  is a parameter of nonlinearity equal to approximately 3.4 for water,

$c_o$  is the speed of sound, and

R is the range.

The source level equation is

$$L_{sd} = -141 + 20 \log F_- - 40 \log \theta_d + 2(L_s - DI) \text{ dB re } 1 \mu\text{bar at 1 yd} . \quad (3)$$

$F_-$  is expressed kiloHertz and  $\theta_d$  is expressed in degrees.

The primary function of fig. 2 is the determination of the primary beamwidths normalized with respect to  $\theta_d$ . For a circular transducer of radius a, the normalized beamwidth is given by

$$\psi_d = 92.5/k_o a \theta_d , \quad (4)$$

and for a rectangular transducer of sides l and m,

$$\psi_y = 163/k_o l \theta_d , \quad (5)$$

and

$$\psi_z = 163/k_o m \theta_d . \quad (6)$$

INSTRUCTIONS: Line up  $\theta_d$  with  $F_0$  and mark axis X. Line up point so marked with piston radius or length of side and read  $\psi_d$  or  $\psi_{y,z}$ . DI is obtained by lining up  $F_0$  with piston radius. For a rectangular find the DI value based on the length of each side, average and subtract 4.97 dB.

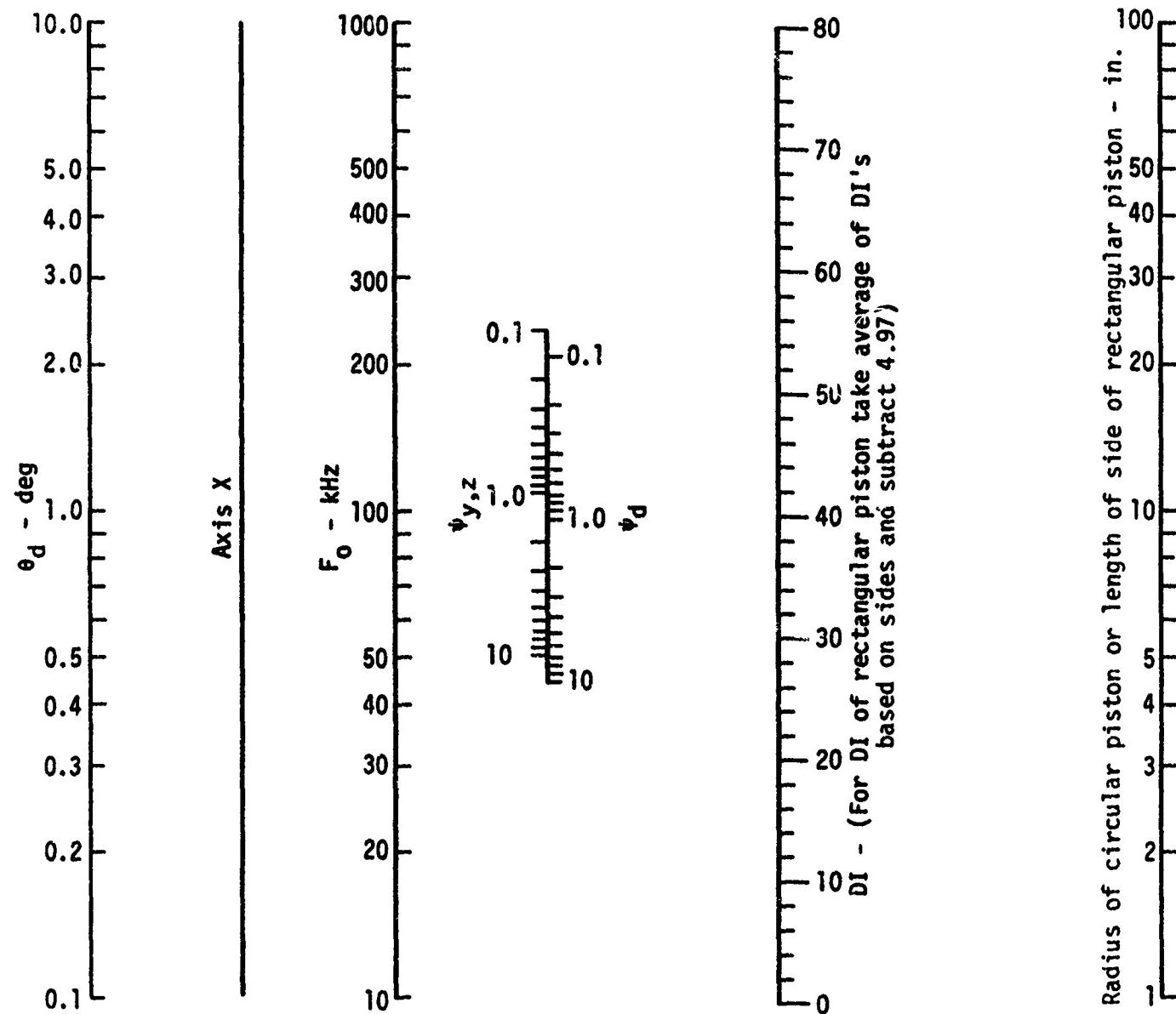


FIGURE 2  
NOMOGRAPH FOR DETERMINING  
NORMALIZED PRIMARY BEAMWIDTHS

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The normalized beamwidths are obtained from fig. 2 by lining up  $\theta_d$  and  $F_o$  to determine the intersection on axis X and then lining up that point with the radius of a circular transducer or the length of a side of a rectangular transducer and reading  $\psi_d$  or  $\psi_{y,z}$ . The DI scale on fig. 2 is provided as a convenience for obtaining the directivity index.

Once the normalized primary beamwidths have been obtained, the source level obtained from fig. 1 may be reduced by the "relative pressure ratio" determined from fig. 3 to give the corrected difference frequency source level. The ratio of the actual difference frequency half beamwidth to  $\theta_d$  may be obtained from figs. 4 and 5. Figures 3 through 5 are reproduced from Berkay and Leahy.

#### Shock Threshold

The Berkay-Leahy theory does not take finite amplitude attenuation into account; hence, it seems reasonable to use the threshold for eventual shock formation as a least upper bound for validity of the results predicted by use of the present nomographs. An approximate model is used to estimate the shock threshold. The piston radiation field is assumed to approximate plane waves out to the Rayleigh distance  $R_o = S/\lambda$ , the piston area divided by wavelength, and spherical waves beyond  $R_o$ . Some unpublished results of Lockwood indicate that shocks will eventually form if

$$\frac{\beta \epsilon k}{\alpha} \left[ 1 - \exp(-\alpha R_o) \right] + \beta \epsilon k R_o \int_{R_o}^{\infty} \exp(-\alpha R) (dR/R) \geq 1 . \quad (7)$$

Figure 6 gives the shock threshold as a function of frequency for several values of  $R_o$ .

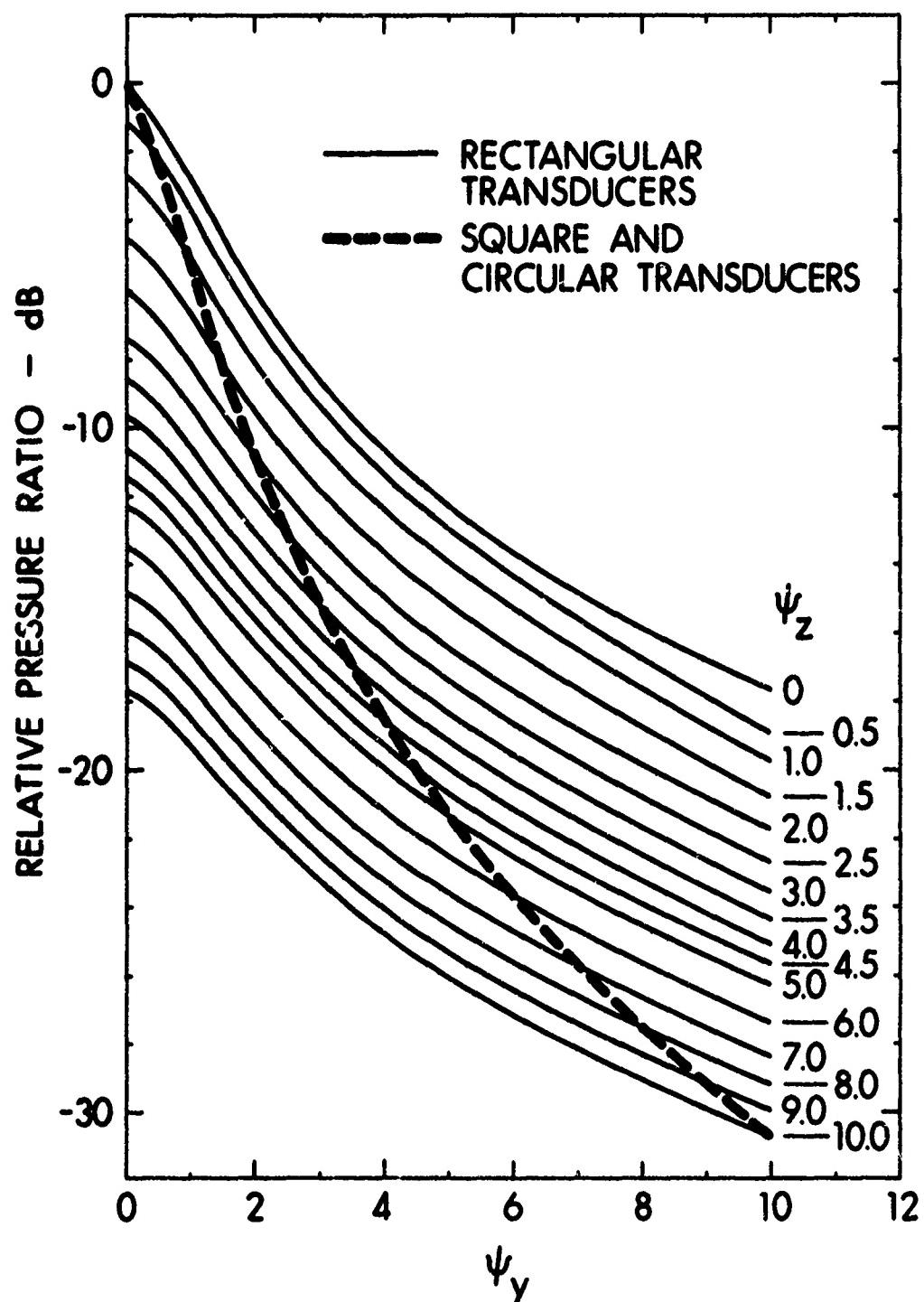
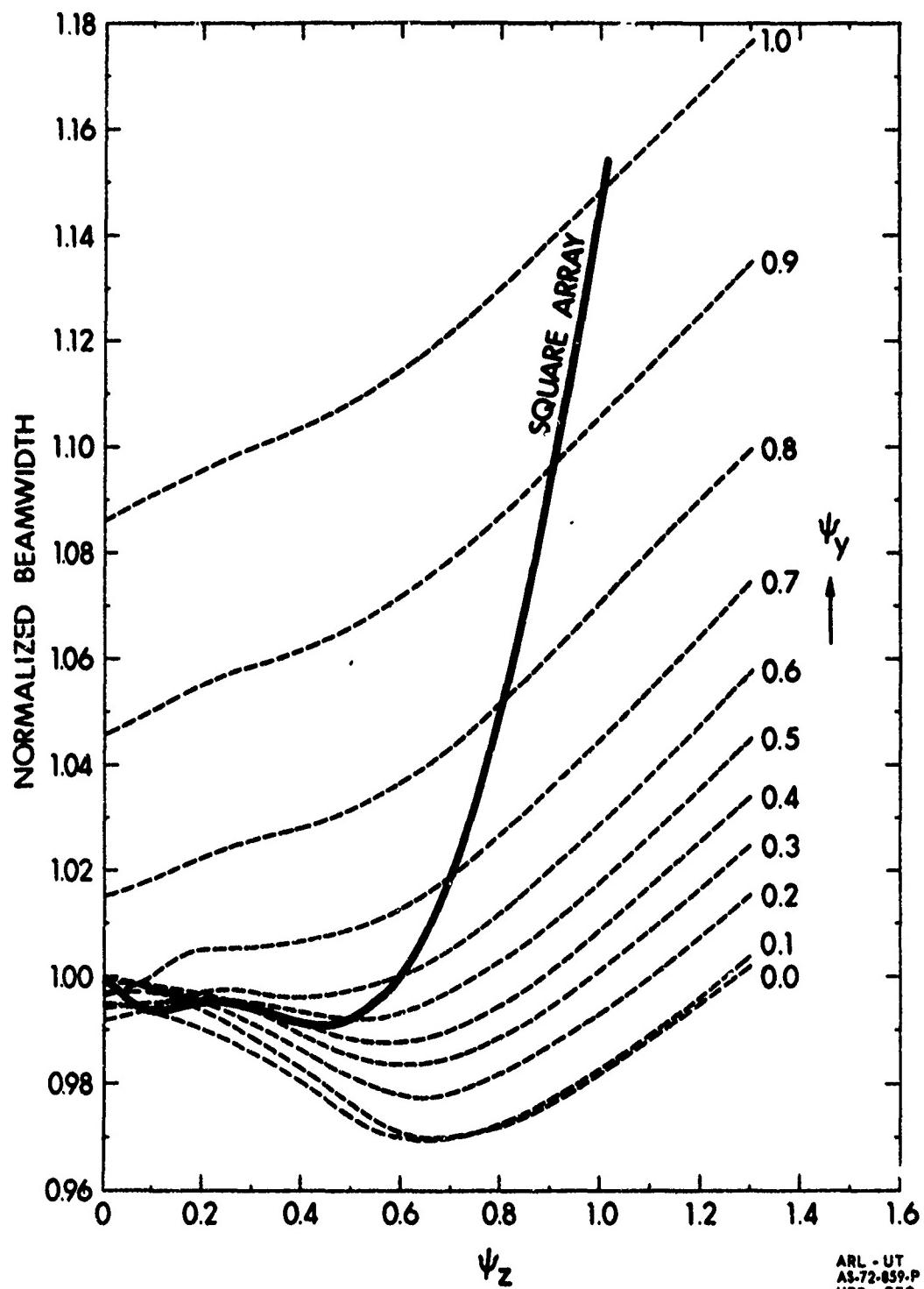


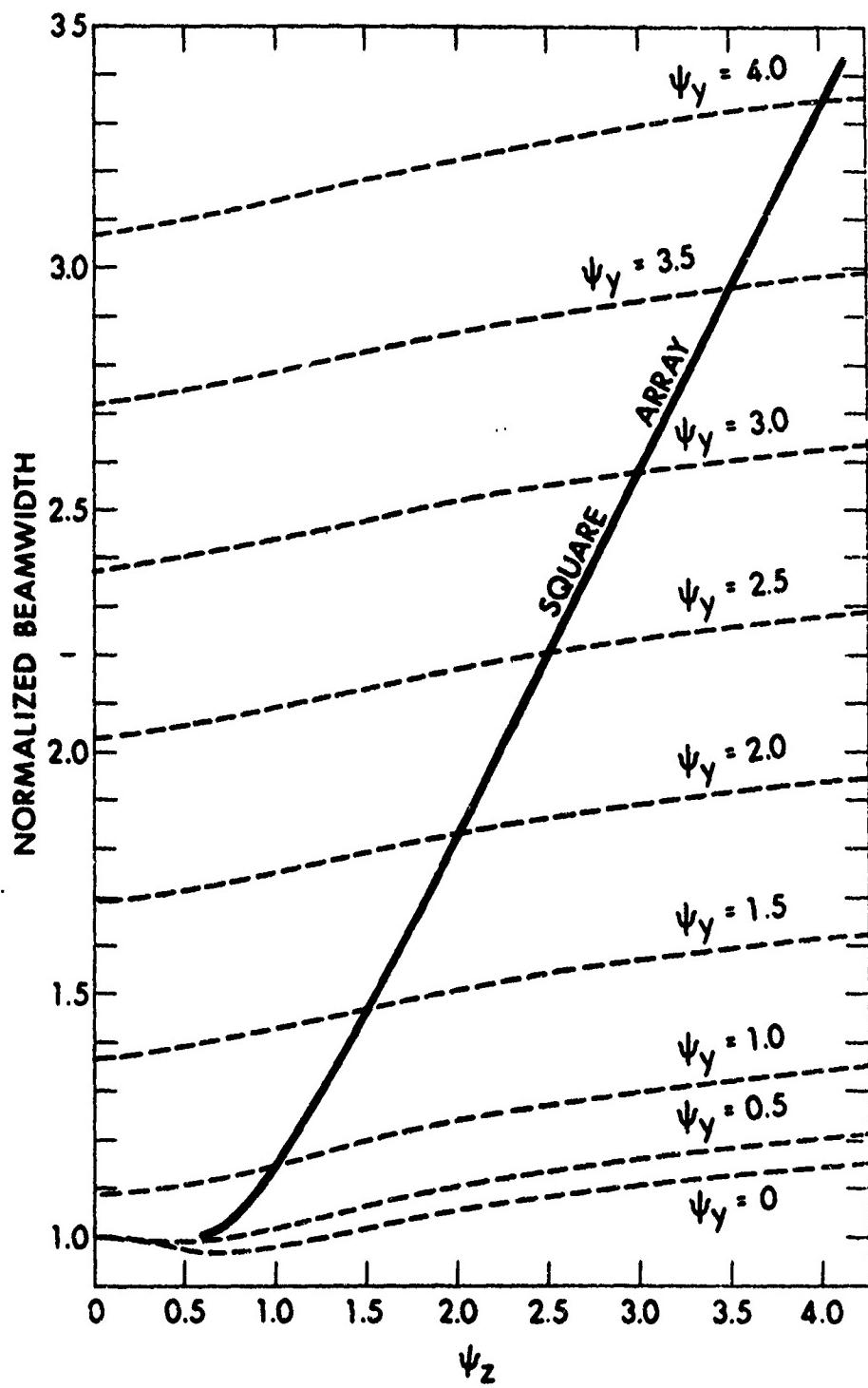
FIGURE 3  
PRESSURE REDUCTION FACTOR,  $|l|$ , FOR  
RECTANGULAR AND CIRCULAR TRANSDUCERS

ARL - UT  
AS-72-858-P  
HOB - RFO  
7-18-72



ARL - UT  
AS-72-859.P  
HOB - RFO  
7-18-72

FIGURE 4  
NORMALIZED HALF-POWER BEAMWIDTHS,  
 $\theta_{HP}/\theta_d$ , FOR RECTANGULAR TRANSDUCERS.  
(THE BEAMWIDTH IN THE x-y PLANE IS SHOWN)



ARL - UT  
AS-72-860-D  
HOB - EJW  
7-18-72

FIGURE 5  
NORMALIZED HALF-POWER BEAMWIDTHS,  
 $\theta_{HP}/\theta_d$ , FOR RECTANGULAR TRANSDUCERS.  
(THE BEAMWIDTH IN THE x-y PLANE IS SHOWN)

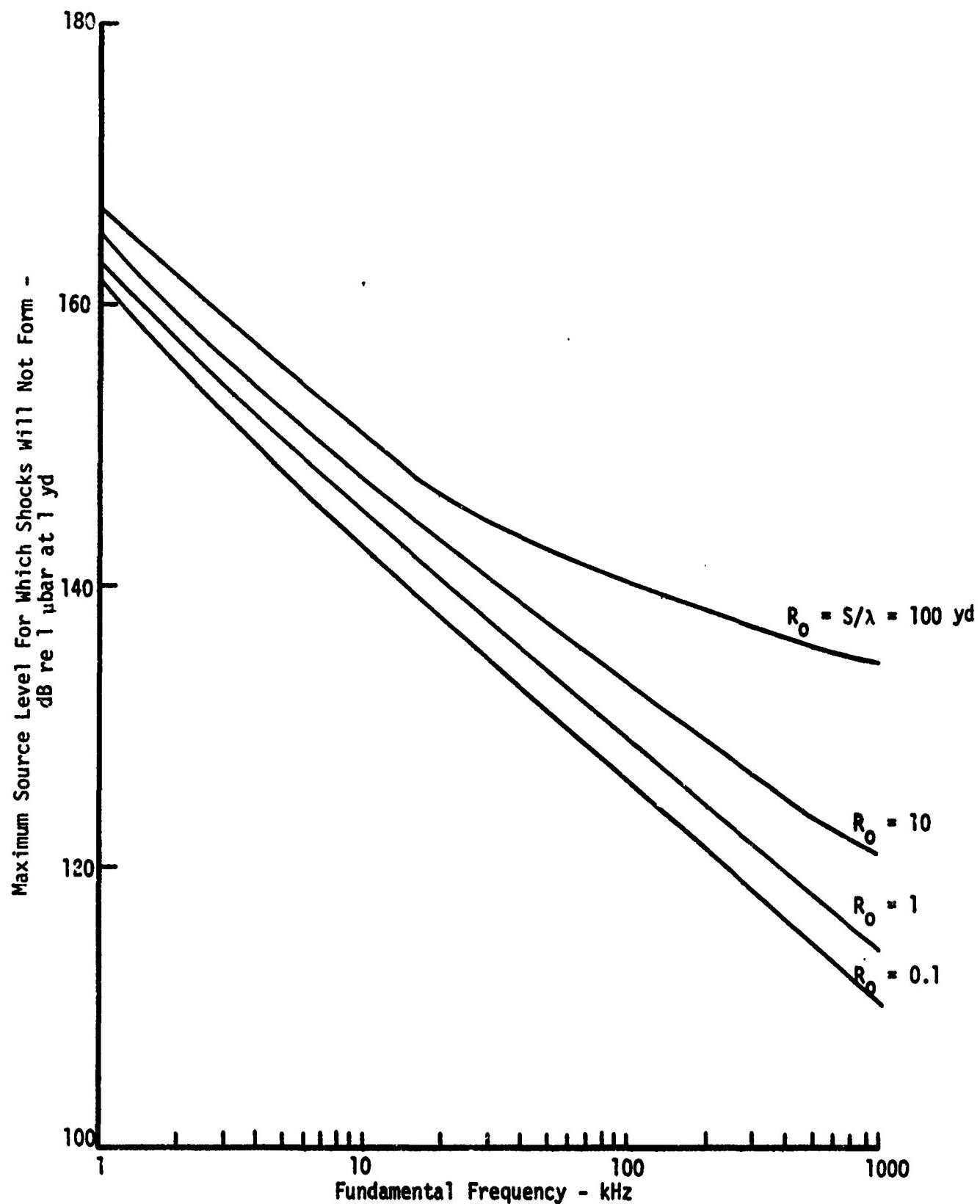


FIGURE 6  
GRAPH OF LEVELS BELOW WHICH SHOCKS WILL NOT FORM  
IN RADIATION FROM A PLANAR PISTON  
IN SEA WATER

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APPENDIX A  
Nomograph for Fresh Water

Figure A-1 is a nomograph for determining the Westervelt source level and beamwidth for fresh water at 20°C. For fresh water calculations it replaces fig. 1.

Figure A-2 is the shock threshold graph for fresh water at 20°C to replace fig. 6.

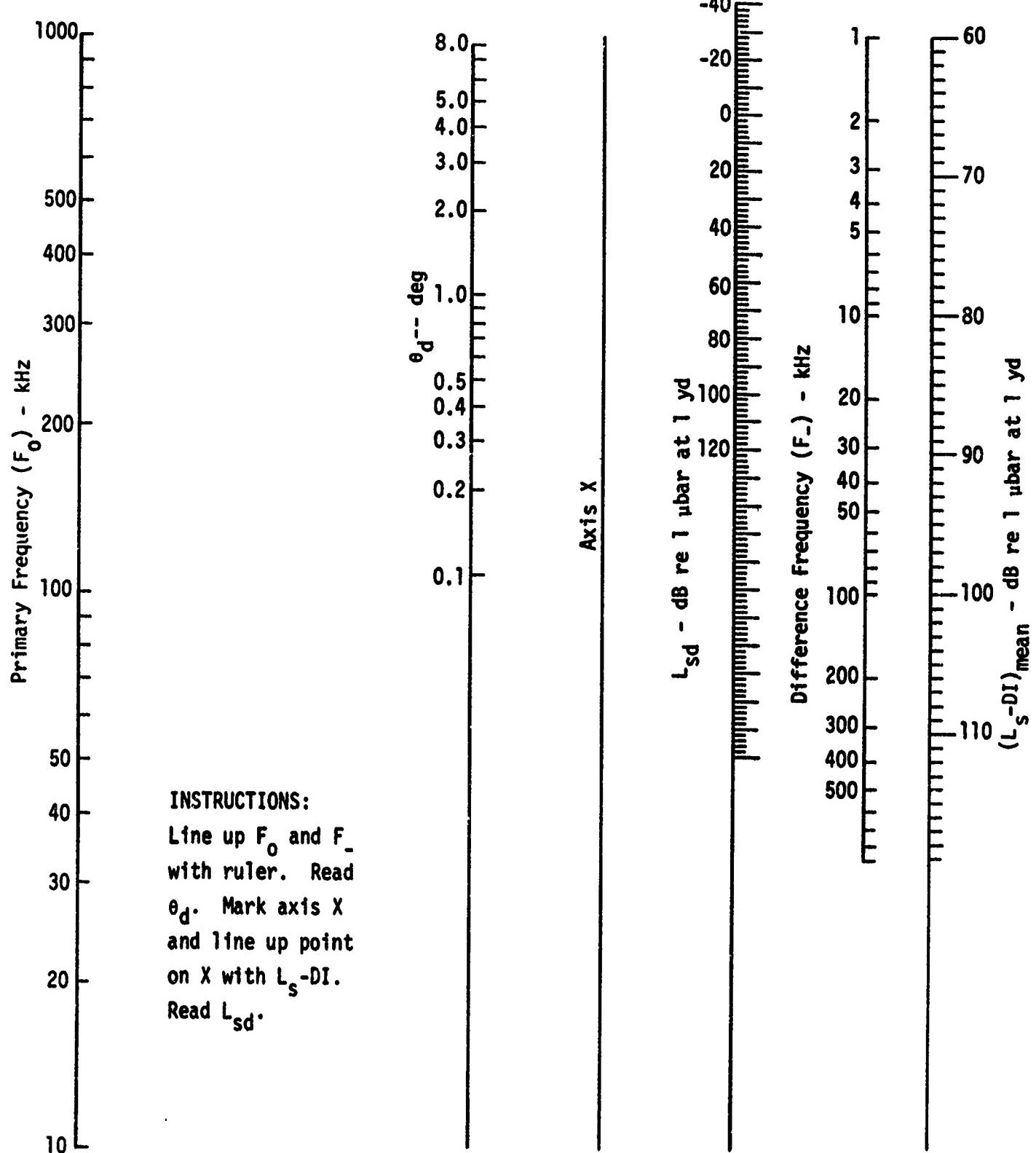


FIGURE A-1  
 NOMOGRAPH FOR DETERMINING  
 DIFFERENCE FREQUENCY SOURCE LEVEL ( $L_s$ ) AND  
 HALF BEAMWIDTH OF A PARAMETRIC ENDFIRE ARRAY  
 IN FRESH WATER (WESTERVELT)

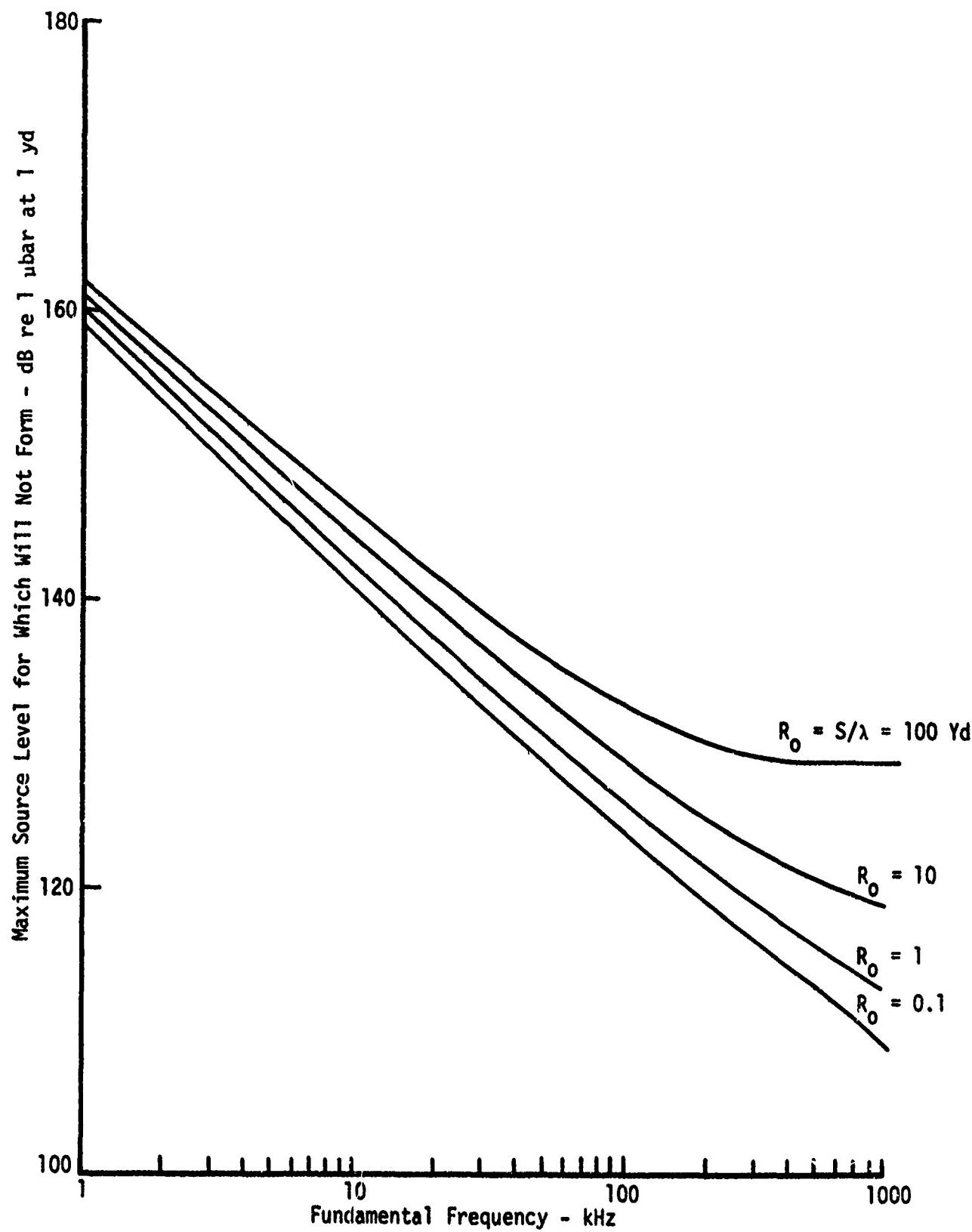


FIGURE A-2  
GRAPH OF LEVELS BELOW WHICH SHOCKS WILL NOT FORM  
IN RADIATION FROM A PLANAR PISTON  
IN FRESH WATER

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1. H. O. Berkay and D. J. Leahy, "Farfield Performance of Parametric Transmitters" (to be published, 1973, Journal of The Acoustical Society of America).
2. R. H. Mellen and M. B. Moffett, "A Model for Parametric Sonar Radiator Design," Naval Underwater Systems Center Technical Memorandum No. PA41-229-71 (1971).
3. P. J. Westervelt, "Parametric Acoustic Array," J. Acoust. Soc. Amer. 35, 535-537 (1963).